Please answer all questions

1. Paul Krugman provides a model of international trade, in which the number of goods that are produced and traded is endogenous ('Increasing Returns, Monopolistic Competition, and International Trade', JIE 1979). A representative consumer's preferences are given by the utility function

$$U = \sum_{i=1}^{n} v(c_i),$$

where c_i is the per-capita consumption level of variety i and $v(\cdot)$ is a strictly concave function. Each firm produces one variety using the identical cost function $l_i = a + bx_i$, where $a, b > 0$, l_i is labor employed, and x_i is output of variety i. The economy has L workers; each worker supplies one unit of labor. The price elasticity of demand for variety i is given by $\eta_i = -v'/(v''c_i)$, where it is assumed that $d\eta_i/dc_i < 0$.

   (a) Derive and explain the equilibrium conditions.

   (b) Use Cramer's Rule to show formally that a marginal increase in L reduces both the relative price, p/w, and per capita consumption, c, of a variety. Then show that the equilibrium number of varieties, n, rises.

   (c) Now suppose that $v(c_i) = c_i^{(\sigma-1)/\sigma}$. Show that the price elasticity of demand is equal to σ, and derive the equilibrium values of p/w, y and n.

   (d) Explain how the effects of an increase in L differ between parts b. and c. What does this imply for the gains from trade?

2. Consider the Heckscher-Ohlin-Vanek Theorem.

   (a) State the HOV Theorem.

   (b) Prove the HOV Theorem, making sure to state each assumption you have to make.

   (c) Now consider an example. Country i's output vector is $Y^i = (3, 2)$, the world output vector is $Y^w = (30, 120)$, the price vector is $p = (4, 4)$, and the matrix of input coefficients is $A = \begin{pmatrix} a_{1L} = 1 & a_{2L} = 2 \\ a_{1K} = 2 & a_{2K} = 1 \end{pmatrix}$.

      - Derive the country's commodity imports and exports.
      - Derive its factor content of trade.
      - What is the equilibrium relative factor price?

3. Consider a Ricardian model with two countries, Home and Foreign, and two industries, light and heavy. The unit input requirements are as follows:

	Home	Foreign
light	$a_l^H = 8$	$a_l^F = 10$
heavy	$a_h^H = 4$	$a_h^F = 2$

Each country has $L = 4000$ workers who each supply one unit of labor. The representative worker in each country has the utility function $U(c_l, c_h) = ln\ c_l + ln\ c_h$, so that the demand for good $i = l, h$ is given by $c_i = wL/(2p_i)$, where w is the wage rate and p_i is the price of good i.

(a) Suppose that the countries are in autarky. For each country, draw the production possibility frontier (PPF), derive the autarky relative price and compute the consumption/production of each commodity.

(b) Which country has a comparative/absolute advantage in which commodity? Explain.

(c) Suppose now that the two countries engage in free trade. Construct the relative supply and the relative demand curve and depict them in a diagram.

(d) Derive the free trade relative price and compute each country's production and consumption of each commodity.