

Please answer all questions

For Questions 1 and 2 consider a version of the Melitz/Ottaviano model. The utility function of an individual consumer is given by

$$U = q_0^c + \int_{i \in \Omega} q_i^c di - \frac{1}{2} \gamma \int_{i \in \Omega} (q_i^c)^2 di - \frac{1}{2} \left(\int_{i \in \Omega} q_i^c di \right)^2.$$

From this utility function we can derive the following market demand faced by firm i :

$$q_i = \frac{L}{N + \gamma} - \frac{L}{\gamma} p_i + \frac{N}{N + \gamma} \frac{L}{\gamma} \bar{p},$$

where N is the mass of firms, L is the number of consumers, $\gamma > 0$ is a parameter that measures the degree of product differentiation between varieties, p_i is the price of variety i , and \bar{p} is the average industry price. Firms entering the market incur a one-time sunk cost f_E . Firms are monopolistically competitive so that they take N and \bar{p} as fixed when making their decisions.

1. Suppose that the marginal cost of production is $0 \leq c \leq 1$ and that c is identical across firms. The country is in autarky.

- (a) Derive the profit-maximizing price of firm i as a function of \bar{p} .
- (b) Note that with identical firms the profit maximizing prices have to be identical across firms, so that for any firm i we have $p_i = \bar{p} = p$. Use this fact to show that the equilibrium price of a firm is equal to $p = c + \frac{\gamma(1-c)}{N+2\gamma}$.
- (c) In the autarky equilibrium with free entry and exit the equilibrium mass of firms is $\hat{N} = (1-c)\sqrt{\gamma L/f_E} - 2\gamma$, the equilibrium price is $\hat{p} = c + \sqrt{\gamma f_E/L}$ and the equilibrium output of a firm is given by $\hat{q} = \sqrt{L f_E/\gamma}$. If two identical countries integrate their markets, explain how this gives rise to a scale and a variety effect. Explain how these effects impact on social welfare.

2. Suppose now that the marginal cost of production c is uniformly distributed on the interval $[0, c_M]$ with density $1/c_M$.

- (a) Show that in autarky the marginal cost at which a firm is just indifferent between remaining in the market and exiting is given by $c_D \equiv \frac{\gamma + N\bar{p}}{\gamma + N}$.
- (b) Derive the price elasticity of demand.
- (c) Show that in autarky the profit maximizing price, output, operating profit of a firm with marginal cost c are given by

$$\begin{aligned} p(c) &= \frac{1}{2} (c_D + c), \\ q(c) &= \frac{L}{2\gamma} (c_D - c), \\ \pi(c) &= \frac{L}{4\gamma} (c_D - c)^2, \end{aligned}$$

- (d) Write down the expected zero-profit condition of a firm. [Hint: no calculations necessary.]
- (e) In the autarky equilibrium the average price, average firm output, and the mass of firms are given by:

$$\begin{aligned}\bar{p} &= \frac{3}{4}c_D, \\ \bar{q} &= \frac{L}{4\gamma}c_D, \\ N &= 4\gamma\frac{1-c_D}{c_D}, \\ \text{with } c_D &= \left(\frac{12\gamma f_E}{L}\right)^{\frac{1}{3}}\end{aligned}$$

If two identical countries integrate their markets, show how this affects the average price and average output, as well as the mass of firms.

- (f) Explain how economic integration affects industry productivity and social welfare and compare your results to those of Question 1.c.
3. Consider the two-good, two-factor model discussed in class, and assume that industry 1 is labor intensive:

$$\frac{a_{1L}}{a_{1K}} > \frac{a_{2L}}{a_{2K}}.$$

- (a) Write down the four equilibrium conditions.
- (b) Explain under which conditions each price vector (p_1, p_2) corresponds to unique factor prices (w, r) . Use a diagram to illustrate ONE case in which these conditions are violated.
- (c) Suppose the conditions in (b) are satisfied. Use Cramer's Rule to show formally how, when product prices are fixed, a marginal increase in the capital endowment ($dK > 0, dL = 0$) affects the outputs of goods 1 and 2.